0.652. When fully turbulent flow (resulting from boundary-layer trips or natural transition) exists over large portions of the body, center-of-pressure location is no longer a strong function of angle of attack (Fig. 1b).

It is of interest to note that the above changes in center-of-pressure location are associated with a yawing moment perturbation (Fig. 2). Presumably such a perturbation arises from an asymmetry in the transition front at small angles of attack.

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Breakdown Condition of an Axisymmetric Swirling Flow

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Introduction

MANY explanations have been proposed for vortex breakdowns of swirling flows, such as finite transition theory by Benjamine,[1] instability theory by Ludvieg,[2] etc. Quite recently Tsai and Widnall[3] reported a group velocity criterion for vortex flow.

On the other hand, one method for prediction and explanation of vortex breakdown using a simple axial velocity profile has been proposed by the author.[4] The principle underlying this method is to find solutions which approximately satisfy the Navier-Stokes equations, thus leading to the critical condition of breakdown. Consequently, a condition of breakdown of the axisymmetric swirling flows that the critical angular velocity ω_c is nearly 1.0 was obtained. When comparing this value with the already obtained experimental data, a great difference between them was not found.[4] In this Note more complicated and extended axial velocity profiles are used, for which expressions for calculating the critical angular velocity are derived, and the critical angular velocity is calculated for some axial velocity profiles characteristic of the swirling flows.

Formulation of the Problem

Several experiments on breakdown of swirling flows have been performed so far[5-7] and referring to those data, in a previous paper[4] a simple function for the axial velocity profile was used; specifically where r and z are the radius and axial length, respectively, in cylindrical coordinates.

$$w = 1 - A(1-\tilde{a}r^2)z(z-2\tilde{b}) \tag{1}$$

where $A = 1/\{L(L-2\tilde{b})\}$, and $\tilde{a}$, $\tilde{b}$, and L are parameters. Each variable is made nondimensional with reference to the core radius and the velocity on the axis at the upstream boundary. In this Note this simple profile is extended to a rather complicated one as follows:

$$w = 1 + az + bz^2 + (c+dz+ez^2+fz^3)(g+hr)r^2 \tag{2}$$

where $a,b,c,d,e,f,g,$ and h are parameters.

Using this axial velocity profile, the radial velocity component u can be calculated directly from the continuity equation.

$$u = -r(a+2bz)/2 - (d+2ez+3fz^2)(g/4+hr/5)r^3 \tag{3}$$

In Eqs. (2) and (3), p_1 and p_2, which are the pressures obtained from the radial and axial equations of motion, respectively, are represented by the following expressions:

$$\begin{aligned} p_1 = & -\{r(a+2bz)/2 + (d+2ez+3fz^2)(g/4+hr/5)r^3\}^2/2 \\ & + (1+az+bz^2)\{b/2+2(e+3fz)(g/16+hr/25)r^2\}r^2 \\ & + (c+dz+ez^2+fz^3)\{b(g/4+hr/5)r^4 + 2(e+3fz) \\ & \times (g^2/24+9ghr/140+h^2r^2/40)r^6\} + \int_0^r \frac{v^2}{r}\,\mathrm{d}r \\ & - \epsilon(d+2ez+3fz^2)(g+hr)r^2 - \epsilon 6f(g/16+hr/25)r^4 \\ & + \{1-(1+az+bz^2)^2\}/2 + \epsilon 4g(c+dz/2 \\ & + ez^2/3+fz^3/4)z + \epsilon 2bz \end{aligned} \tag{4}$$

and

$$\begin{aligned} p_2 = & \, r^2z(2g+3hr)\{ac + (ad+2bc)z/2 + (ae+2bd)z^2/3 \\ & + (af+2be)z^3/4 + 2bfz^4/5\}/2 \\ & + (g/4+hr/5)r^4z(2g+3hr)\{cd + (2ec+d^2)z/2 \\ & + (fc+de)z^2 + (2df+e^2)z^3/2 + efz^4 + f^2z^5/2\} \\ & - [\{1+az+bz^2 + (c+dz+ez^2+fz^3)(g+hr)r^2\}^2 \\ & - \{1+c(g+hr)r^2\}^2]/2 \\ & + \epsilon(4g+9hr)z(c+dz/2+ez^2/3+fz^3/4) \\ & - \epsilon 6fz(g/4+hr/5)r^3 - \{ar/2 + d(g/4+hr/5)r^3\}^2/2 \\ & + \{b/2+2e(g/16+hr/25)r^2\}r^2 + c\{b(g/4+hr/5) \\ & + 2e(g^2/24+9ghr/140+h^2r^2/40)r^2\}r^4 \\ & + \omega^2r^2/2 - \epsilon d(g+hr)r^2 - \epsilon 6f(g/16+hr/25)r^4 \end{aligned} \tag{5}$$

where $\epsilon \equiv Re^{-1}$.

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Table 1 Values of parameters in three cases calculated in the present analysis

Cases	Parameters a	b	c	d	e	f	g	h
A	−0.02083	−0.1042	−0.02	0.02	0.1607	0	1.0	0
B	−0.02083	−0.1042	0	0.02083	0.1042	0	1.0	0
C	−0.02083	−0.1042	0.02	0.02	0.1022	0	1.0	0

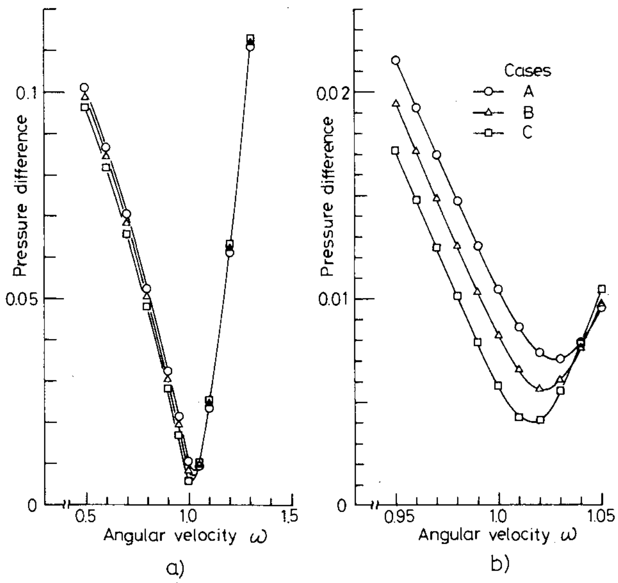

Fig. 1 Variations of pressure difference for each value of ω; – ○ –: case A, – △ –: case B, – □ –: case C.

Approximate circumferential velocity component v, at any point in the domain considered, now is calculated by the nonviscous theory, that is, the circulation function K, which is defined as $K \equiv vr$, has the same value on the same stream surface. The stream surface function ψ is expressed at $z=0$ as

$$\psi(r,0) = \int_0^r w(r,0)r\mathrm{d}r = r^2/2 + c(g/4 + hr/5)r^4 \tag{6}$$

The circulation function K is expressed by assuming solid rotation at $z=0$ in the following form:

$$K(r,0) = \omega r^2 \tag{7}$$

The relation between K and ψ is obtained from Eqs. (6) and (7).

$$\psi = (K/\omega)\{ 1/2 + (cg/4)K/\omega + (ch/5)(K/\omega)^{3/2} \} \tag{8}$$

On the other hand, the stream surface function at any point in the domain examined is calculated using Eq. (2) as follows:

$$\psi = \int_0^r wr\mathrm{d}r = (r^2/2)(1 + az + bz^2)$$
$$+ (c + dz + ez^2 + fz^3)(g/4 + hr/5)r^4 \tag{9}$$

Substituting Eq. (9) into Eq. (8), the following nonlinear equation for K is obtained:

$$(K/\omega)\{ 1/2 + (cg/4)K/\omega + (ch/5)(K/\omega)^{3/2} \}$$
$$= (r^2/2)(1 + az + bz^2)$$
$$+ (c + dz + ez^2 + fz^3)(g/4 + hr/5)r^4 \tag{10}$$

Table 2 Values of pressures on r-z plane, case C, where $\omega = 1.02$ and p_1 and p_2 are arranged in the upper and lower rows, respectively

z	r = 0	0.1	0.2	0.3	0.4
0.0	0	0.004682	0.01874	0.04222	0.007520
0.75	0.07145	0.07556	0.08796	0.1087	0.1381
		0.07543	0.08740	0.1074	0.1355
1.5	0.2308	0.2332	0.2405	0.2529	0.2708
		0.2332	0.2405	0.2526	0.2695
2.25	0.4111	0.4115	0.4130	0.4159	0.4206
		0.4117	0.4137	0.4166	0.4200
3.0	0.5042	0.5037	0.5022	0.4997	0.4966
		0.5040	0.5030	0.5004	0.4948

This equation bears the characteristic that K need not be calculated for each value of ω. That is, defining $\bar{K}$ by $\bar{K} = K/\omega$, $\bar{K}$ is calculated only once when the axial velocity function is determined. Thereafter K is calculated from the relation that $K = \omega\bar{K}$ for each value of ω. The solutions of this equation were sought numerically using the Newton method.

From the aforementioned, we consider the approximate solutions to exist when the difference between p_1 and p_2 takes a minimum value, varying the value of ω. The pressure difference was estimated by the following expression which is the norm at some grid points[4]:

$$D(p_1 - p_2) = \left[\sum_{i=2}^{5} \sum_{j=2}^{5} \{ p_1(r_j, z_i) - p_2(r_j, z_i) \}^2 \right]^{1/2} \tag{11}$$

Results and Discussions

The calculation region extends radially from 0 to 0.4 and axially from 0 to 3.0, and a value of ϵ is taken as $1.160E-3$. Three examples characteristic of swirling flows are calculated by the present method, that is, the radial profiles of the axial velocity component at $z=0$ have negative, zero, and positive curvatures, respectively, which are named cases A, B, and C, respectively (Table 1).

The variations of the pressure difference calculated by Eq. (11) for various values of ω are shown in Fig. 1. As shown in Fig. 1a, the difference takes sharp minimum values when the value of ω is nearly equal to 1.0 in all three cases. In particular, it should be noted that the difference reaches a value of $4.129E-3$ in case C when $\omega = 1.02$. The values of p_1 and p_2 in this case are shown in Table 2. In case A a value of $7.077E-3$ is represented as a minimum when $\omega = 1.03$. As shown in Fig. 1b, ω_c becomes relatively larger in the case of negative curvature, although the minimum value of difference increases. From the fact that the experimental profile has a negative curvature and the angular velocity has a value of 1.17,[7] it would appear that it is reasonable and plausible for the critical angular velocity in this case to increase.

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Estimation of Heat-Transfer Coefficient in a Rocket Nozzle

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Nomenclature

a,b,c,d	= coefficients of square temperature matrix
h	= heat-transfer coefficient
k	= thermal conductivity of material
L	= thickness of material
$\dot{q}_c$	= surface heat flux
T	= temperature
T^{n-1}	= temperature at beginning of time step
T^n	= temperature at end of time step
T_g	= combustion gas temperature
T_0	= temperature at surface
t	= time
Δt	= computing time
x	= space coordinate
Δx	= node thickness
α	= thermal diffusivity of material
Subscripts	
$i,0,1$	= node identifier
$0,r$	= surface
j	= thermocouple location
Superscript	
n	= designated the point of time $t+\Delta t$

Introduction

THE determination of the temperature distribution in a rocket nozzle wall subjected to a high-temperature and high-heat-flux environment requires the knowledge of the total heat transferred from the combustion gases. The dominant mode of energy transport in chemical rocket engines is by convection. It is therefore important to estimate accurately the convective heat transfer to the wall in order to achieve an optimum thermal protection system. In heat-transfer studies, many experimental difficulties may arise in implanting heat-flux sensors or thermocouples at the surface for heat-transfer measurements. Furthermore, the presence of a probe at the surface disturbs the condition of the boundary and the flow process adjacent to it and thus actual wall heat flux. In these circumstances it is therefore desirable that the prediction of surface temperature and heat flux be accomplished by inverting the temperature as measured by a probe located interior to the surface of the solid material. Such a problem is termed the inverse problem.

Problems of this kind can be solved using an exact solution,[1] an integral method,[2-5] or a finite-difference method.[6-8] The method sufficiently powerful to solve the general problem appears to be latter, although the basic concepts can be applied to the integral method.

The present Note reports an iterative scheme to obtain values of surface temperature, surface heat flux, heat-transfer coefficient, and combustion gas temperature using a numerical technique in conjunction with the measured temperature history at the outer surface of the rocket nozzle.

Analysis

The physical problem involves a convectively heated slab of finite thickness having a heat sink at one surface and a perfect insulation at the other.

If the temperature on the boundary (surface) node receiving the surface heat can be bounded, then all interior code temperatures will also be included. This boundness must prevent wild oscillation in the surface temperature. This can be achieved by using implicit representation at the surface as

$$\frac{\dot{q}_c}{\Delta x k} - \frac{(T_0^n - T_1^n)}{\Delta x^2} = \frac{1}{2\alpha}\frac{(T_0^n - T_0^{n-1})}{\Delta t} \tag{1}$$

where the subscripts 0 and 1 denote the node identifiers and the superscript n indicates that the value is taken at time $t+\Delta t$.

Since only one-dimensional heat transfer is being considered, the solution can be obtained by solving the tridiagonal system of equations

$$a_i T_{i-1}^n + b_i T_i^n + c_i T_{i+1}^n = d_i \quad \text{for } 0 \le i \le r \tag{2}$$

Rearranging Eq. (1) into this tridiagonal form

$$\left[-2 - \frac{\Delta x^2}{\alpha \Delta t}\right] T_0^n + 2T_1^n + \frac{2\dot{q}_c \Delta x}{k} + \frac{\Delta x^2}{\alpha \Delta t} T_0^{n-1} = 0 \tag{3}$$

The coefficients in Eq. (2) may be readily obtained as

$$a_0 = 0,\ b_0 = -2 - \frac{\Delta x^2}{\alpha \Delta t},\ c_0 = 2, \text{ and } d_0 = -\frac{2\dot{q}_c \Delta x}{k} - \frac{\Delta x^2}{\alpha \Delta t} T_0^{n-1} \tag{4}$$

The tridiagonal system of equations can be solved using the Thomas algorithm.[9] But in the foregoing Eq. (3), $\dot{q}_c$ is an unknown parameter, thus the solution of the complete problem from $x=0$ to x_j cannot be obtained readily because the boundary condition is not known at $x=0$, but rather an interior temperature history is given. In estimating one minimizes

$$F(\dot{q}_c) = [T_c(x_j,t) - T_m(x_j,t)] \tag{5}$$

where T_c and T_m are, respectively, calculated and measured thermocouple temperatures at (x_j,t).

The calculated temperature is, in general, a nonlinear function of $\dot{q}_c$. A simple procedure approximates at each iteration the calculated temperatures by the Newton-Raphson

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